Int. J. Heat Mass Transfer. Vol. 20, pp. 1090–1092. Pergamon Press 1977. Printed in Great Britain

ATMOSPHERIC DISPERSION OF AN INSTANTANEOUS POLLUTANT RELEASE

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(*Received* 28 *July* 1976 *and in revised form* 13 *October* 1976)

NOMENCLATURE

C, local concentration of pollutant;
D_y, D_z, turbulent eddy diffusivities in the y and z directions;
D_2, $\dfrac{D_y}{D_z}$;
H, inversion height above the ground;
M, total mass of pollutant released;
t, time;
U, dimensionless downwind velocity, $U = \dfrac{u}{u_0}$;
u, downwind velocity in the x-direction;
u_0, velocity at the inversion;
X, dimensionless downwind coordinate, $X = \dfrac{xD_z}{H^2u_0}$;
x, downwind coordinate;
Y, dimensionless transverse coordinate, $Y = \dfrac{y}{H}$;
y, transverse coordinate;
Z, dimensionless vertical coordinate, $Z = \dfrac{z}{H}$;
Z_1, dimensionless height of the source;
z, vertical coordinate;
z_1, height of the source.

Greek symbols

δ, Dirac delta function;
θ, dimensionless local concentration, $\theta = \dfrac{H^4u_0C}{MD_z}$;
τ, dimensionless time, $\tau = \dfrac{D_zt}{H^2}$.

INTRODUCTION

THE FACTORS which govern the transport of a pollutant in the atmosphere are fairly complex. Efforts to model such transport may range from exact solutions of the primitive conservation equations for mass, energy, and momentum to semi-empirical treatments signified by the eddy diffusion approach or the Gaussian model. Ultimately, it is expected that the best and most useful results will emerge from the exact solutions of the conservation equations; however, even in relatively simple problems, this requires the use of finite difference procedures demanding substantial amounts of computing resources (see [1], for example). In the other extreme, the Gaussian models; which enjoy wide popularity in view of their simplicity, are limited in scope. For instance, they are inadequate in *properly* accounting for the effect of vertical wind shear. In the case of the instantaneous source, the Gaussian approach as recommended by Turner [2] assumes the standard deviation of the distribution to be the same in the axial and crosswind directions; this would result in a substantial underprediction of axial dispersion due to wind shear. Eddy diffusivity models, while admittedly semi-empirical, allow for the direct incorporation of realistic effects such as wind shear and transient phenomena in a natural fashion. They have been used successfully to describe atmospheric transport (see for example [3–6]). In this work, the transport of a gaseous pollutant released into the atmosphere over a relatively short period of time due to an accident will be simulated using the eddy diffusivity model. The model incorporates a persistent inversion at a height H above ground. In the general case of a wind profile which depends both on position and time and eddy diffusivities which depend on position, the model equations have been solved analytically in [7] using generalized dispersion theory which was developed in [8–9] and applied to atmospheric transport in [3, 4]. Here, we shall only present illustrative results for the case of a linear wind profile (couette flow) and constant eddy diffusivities. The transport of pollutant by eddy diffusion in the downwind direction will be neglected in comparison to transport by advection. These simplifying assumptions still permit one to retain the transport mechanisms of interest, and allow the computation of the results from the theory with ease. Under the assumptions, the local pollutant concentration $C(t, x, y, z)$ will satisfy the convective diffusion equation

$$\frac{\partial C}{\partial t} + u(z)\frac{\partial C}{\partial x} = D_y\frac{\partial^2 C}{\partial y^2} + D_z\frac{\partial^2 C}{\partial z^2} \tag{1}$$

where $u(z) = u_0(z/H)$ is the downwind velocity. For the release of pollutant of mass M instantaneously at time zero at $(0, 0, z_1)$, the initial condition on C is

$$C(0, x, y, z) = M\delta(x)\delta(y)\delta(z - z_1). \tag{2a}$$

Since the ground is impermeable to the pollutant,

$$\frac{\partial C}{\partial z}(t, x, y, 0) = 0 \tag{2b}$$

while assuming an impervious inversion lid at $z = H$ results in

$$\frac{\partial C}{\partial z}(t, x, y, H) = 0. \tag{2c}$$

Because of the finiteness of the pollutant distribution,

$$C(t, \infty, y, z) = 0 \tag{2d}$$

$$C(t, x, \pm\infty, z) = 0. \tag{2e}$$

Equations (1) and (2) may be written in dimensionless form as follows:

$$\frac{\partial\theta}{\partial\tau} + U(Z)\frac{\partial\theta}{\partial X} = D_2\frac{\partial^2\theta}{\partial Y^2} + \frac{\partial^2\theta}{\partial Z^2} \tag{3}$$

with the conditions

$$\theta(0, X, Y, Z) = \delta(X)\delta(Y)\delta(Z - Z_1) \tag{4a}$$

$$\frac{\partial\theta}{\partial Z}(\tau, X, Y, 0) = 0 \tag{4b}$$

$$\frac{\partial\theta}{\partial Z}(\tau, X, Y, 1) = 0 \tag{4c}$$

$$\theta(\tau, \infty, Y, Z) = 0 \tag{4d}$$

$$\theta(\tau, X, \pm\infty, Z) = 0. \tag{4e}$$

Here $U(Z) = Z$.

The solutions of equations (3) and (4) may be obtained by separating the Y dependence followed by the application of generalized dispersion theory in a manner very similar to that illustrated in [3–4]. Since the details are, for the most part, identical to those reported in [4], they will be omitted here. The interested reader may consult [7] for the actual details of the solution. From the analytical solution in [7], the dimensionless pollutant concentration θ may be calculated as a function of dimensionless time τ, and the dimensionless position coordinates X, Y and Z. It may be seen from equations (3) and (4) that θ also depends parametrically on the dimensionless source height Z_1 and the ratio of transverse to vertical diffusivity, D_2.

In a distributed wind, the dispersion of pollutant occurs due to a complex interaction between the mechanisms of downwind convection and eddy diffusion. For a ground level release, the material moves downwind extremely slowly at small times due to the very small velocities prevailing near the ground. So, for such times, pollutant transport occurs essentially in the vertical and transverse directions due to turbulent diffusion. As time passes, the material which has moved to higher elevations experiences greater convection and also undergoes transverse diffusion. The wind shear causes a differential horizontal transport at higher elevations and establishes vertical concentration gradients at downwind stations, and therefore, results in the

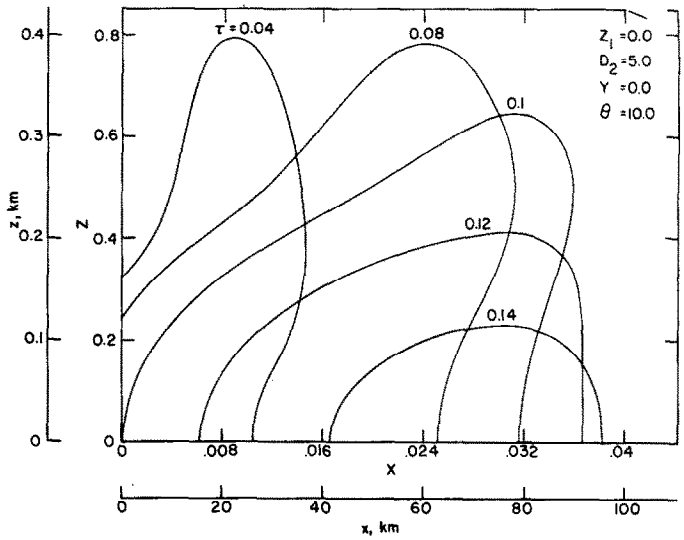

FIG. 1. Isopleths of $\theta = 10.0$ at several values of time; $Z_1 = 0$, $D_2 = 5$, $Y = 0$, $H = 500$ m, $u_0 = 10$ m/s, $D_z = 1$ m^2/s.

RESULTS AND DISCUSSION

Detailed numerical results for $\theta(\tau, X, Y, Z)$ were calculated on an IBM 360/44 for various values of Z and D_2 in order to simulate different source release heights and stability conditions. It was assumed in the calculations that 10 tons of pollutant material were released. Values of $D_2 = 5$ and 2 were used to simulate stable and unstable atmospheric conditions respectively. Only an illustrative sample of the calculated results will be presented here. More detailed results and their interpretation may be found in [7]. The results here are reported in the plane $y = 0$ since the analytical solution in [7] shows that the transverse concentration distribution is Gaussian with the maximum value always occurring at $y = 0$. Also, all the results presented here are for the stable case $D_2 = 5$. It was found in [7], as expected, that unstable conditions always resulted in better mixing of the pollutant in the atmosphere.

Ground level source

Consider a ground level release of pollutant ($Z_1 = 0$) under stable atmospheric conditions. A typical set of parameters would be

$$H = 500\,\text{m}, \quad u_0 = 10\,\text{m/s}, \quad D_z = 1\,\text{m}^2/\text{s}.$$

For this set of parameters, $\theta = 10$ corresponds to a concentration of 160 μg/m^3 which is a typical upper limit of concentration for acceptable air quality for many pollutant materials [10]. Hence, one may assume the isopleth $\theta = 10$ to mark the boundary of the pollutant cloud. Figure 1 shows the development of this isopleth with time in the plane $Y = 0$. diffusion of material toward and away from the ground at such locations. Material diffusing up is convected downwind faster by larger velocity wind; and the material diffusing down is slowed down by smaller velocity wind, resulting in the beginning of buildup of concentration near the ground at downwind stations. From the isopleths at $\tau = 0.04$ (2.778 h) and $\tau = 0.08$ (5.556 h) in Fig. 1, one can see that at the cloud front, material is being transferred toward the ground and the ground level concentration increases with time at farther downwind locations, while at the rear of the cloud, the ground level concentration decreases due to diffusion away from the ground. It is seen from the figure that as time increases, the pollutant cloud moves downwind and experiences dilution as indicated by the isopleth at $\tau = 0.14$ (9.722 h).

In [7], it was found that the dimensionless concentration at the inversion in this case never exceeded 7. If $\theta = 10$ represents the boundary of the cloud, this indicates that the inversion has little influence on pollutant dispersion for this set of parameters.

Slightly elevated source

In the case considered here, the pollutant release height is assumed to be 50 m. All the other parameters are the same as in the case of the ground level source considered earlier.

$$H = 500\,\text{m}, \quad u_0 = 10\,\text{m/s}, \quad D_z = 1\,\text{m}^2/\text{s}.$$

This type of elevated emission will occur, for instance, in the physical situation of the blow-out of elevated gas tanks in industries.

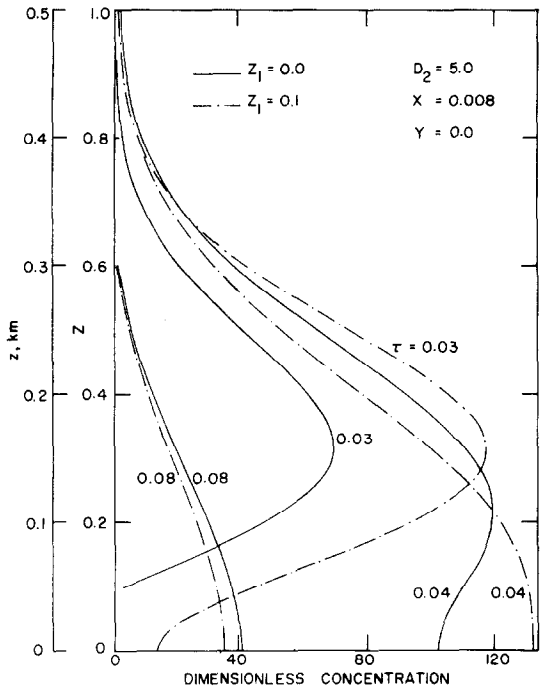

FIG. 2. Dimensionless concentration as a function of elevation for ground level and elevated sources at several values of time; $D_2 = 5$, $X = 0.008$, $Y = 0$, $H = 500$ m, $u_0 = 10$ m/s, $D_z = 1$ m^2/s; $Z_1 = 0$ and 0.1

The vertical concentration profiles for the ground level and the slightly elevated sources are compared in Fig. 2 at several values of time at a downwind location, $X = 0.008$ (20 km). At $\tau = 0.03$ (2.083 h), it is seen that the concentrations throughout the mixing layer are much greater for the slightly elevated source than those for the ground level source, which underscores the increased rate of downwind transport for the elevated emission of pollutant. At $\tau = 0.04$ (2.778 h), it appears that material still is arriving near the ground for the ground level source, whereas the ground level concentration is peaking out for the elevated source. It is interesting to note that at this time, the elevated source results in a greater ground level concentration at this downwind station. However, for greater values of time, the ground level concentration reaches comparable values for the ground level release (not shown in the figure). Beyond this time, concentrations for the elevated source are always smaller than those for the ground level source at any given time.

Source near inversion

A sudden burst of pollutant from a tall chimney may be modeled as an instantaneous source. This is the next case considered here. The height of release of pollutant is assumed to be 200 m and the inversion is assumed to be at 250 m resulting in a dimensionless source height, $Z_1 = 0.8$. For the following discussion, the parameters are specified as

$$H = 250\,\text{m},\quad u_0 = 5\,\text{m/s},\quad D_z = 1\,\text{m}^2/\text{s}.$$

Figure 3 shows the vertical concentration profiles in the plane $y = 0$ as a function of time at a downwind distance $X = 0.04$ (12.5 km) for this case. For small values of time ($\tau = 0.04, 0.05$) the pollutant arrives at this station only at higher elevations near the inversion because of the large velocities present near the inversion. As time passes, concentrations build up near the ground, and the entire mixing layer is used for the dispersion of the material. After a sufficient length of time, concentrations at higher elevations decay faster compared to concentrations near the ground.

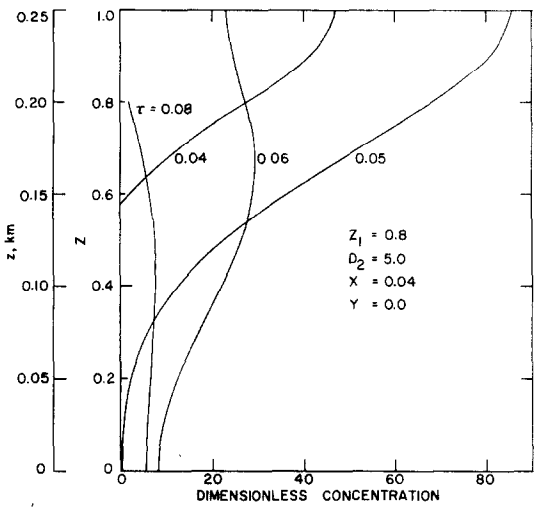

FIG. 3. Dimensionless concentration as a function of elevation for source near inversion at several values of time; $Z_1 = 0.8$, $D_2 = 5$, $X = 0.04$, $Y = 0$, $H = 250$ m, $u_0 = 5$ m/s, $D_z = 1$ m^2/s.

In any case, it may be seen that the ground level concentrations at this location at any time do not exceed 10% of the maximum concentration reached at the inversion. Material released near the inversion travels a long distance and undergoes dilution due to wind shear and eddy diffusion before appreciable concentrations are built up at the ground.

In summary, the sample results shown here illustrate the interaction among the physical mechanisms of pollutant transport and demonstrate the potential of generalized dispersion theory in modeling problems of this kind. For more details, the interested reader is referred to [3, 4, 7–9].

Acknowledgements—The authors would like to thank Professor R. J. Nunge for helpful discussions.

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